Is Real Interest Rate Risk Priced? Theory and Empirical Evidence

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Abstract

We propose a model in which real interest rates respond to both expected consumption growth and time preferences. Exposures to future consumption growth and time preference interest rate shocks are both priced relative to the Capital Asset Pricing Model (CAPM) and the Consumption Capital Asset Pricing Model (CCAPM). However, the two types of interest rate risk have different prices, and when elasticity of intertemporal substitution (EIS) is greater than one, the prices have opposite signs. Moreover, the premia for time preference risk are arbitrarily large when EIS is close to 1. We interpret this as evidence that Epstein-Zin utility with time varying taste shocks and EIS close to 1 implies implausible aversion to future time preference shocks. Empirically, we find little evidence that interest rate risk is priced in the cross-section of stocks. Market equity returns and treasury bonds have only small exposures to real interest rate risk.

1 Introduction

Are expected returns related to covariance with shocks to the real risk free interest rate? Put differently, is the real risk free rate a priced state variable? Since Fama (1970), financial economists have understood that state variables can be priced if they are correlated with changes to (1) investor preferences or (2) the consumption-investment opportunity set.[1] Because the risk free rate is an equilibrium outcome that is sensitive to preferences and consumption-investment opportunities, it is a prime candidate to be a priced state variable.

Previous research primarily focuses on shocks to consumption-investment opportunities. For example, Merton (1973) Intertemporal Capital Asset Pricing Model (ICAPM) considers changing investment opportunities while holding preferences constant. Campbell (1993) follows the same approach to derive ICAPM pricing as a function of changes to expected returns. More recently, Bansal and Yaron (2004) initiated a literature on long-run consumption growth shocks in which expectations about future consumption growth are priced. In these frameworks, positive interest rate shocks are generally good news, which makes long-duration assets valuable hedges, reducing their risk premia.

In contrast, Albuquerque et al. (2016) (AELR) present an interesting model that considers preference shocks to investor patience. In their framework, positive interest rate shocks stem from impatience and are generally bad news, making long-duration assets more risky and increasing their risk premia. We examine a generalized version of the AELR model with both consumption-investment and preference shocks. Expected consumption growth and time preferences both impact interest rates, and covariance with these shocks is priced relative to the Capital Asset Pricing Model (CAPM) and the Consumption CAPM (CCAPM). However, the two types of interest rate risk carry different prices. Relative to both the CAPM and CCAPM, the price of interest rate risk associated with time preference shocks differs from the price of consumption growth interest rate risk by a factor of $\frac{-1}{\psi-1}$, where ψ is elasticity of intertemporal substitution. For $\psi > 1$, this means the two different interest rate risk premia have opposite signs. The current AELR model specification is undefined as ψ approaches 1 and time preference risk premia are very large when ψ is close to 1. This is an important point as we try to understand the true nature of this parameter. For example

[1] Fama (1970) considered consumption and investment opportunities separately. In practice, these two opportunity sets are typically collapsed by considering a single homogeneous consumption good.

Hall (1988) argues that $\psi \approx 0$ while Guvenen (2006) notes that much of the macro literature has concluded that$\psi \approx 1$. Unless there is a well micro-founded reason to explicitly exclude the possibility of $\psi = 1$, we suggest it is beneficial to allow this parameter to take on the full range of values and be dictated by data.

Empirically, we estimate real interest rate shocks based on a vector autoregression (VAR) model of nominal interest rates, CPI inflation rates, and other state variables. When sorted based on interest rate exposure, stocks with high exposure have slightly lower expected returns, both on an absolute basis and relative to CAPM and Fama and French (1993) three factor model predictions. This evidence is consistent with risk premia required for time preference shocks and at odds with risk premia demanded for consumption-investment shocks. That said, the effects are modest, and the return differences are not statistically significant.

Moreover, the overall stock market appears to have very little exposure to interest rate risk. The market's interest rate news beta is an insignificant 0.11, which would carry a risk premium of -8 bps based on our cross-sectional pricing results. This evidence contradicts the conclusions of AELR, who claim that interest rate risk (valuation risk) explains the equity premium puzzle. The main difference between our empirical work and theirs is that we directly estimate covariance between excess returns and real interest rate shocks, whereas AELR do not estimate this moment in their GMM analysis. AELR's benchmark estimates imply that excess equity returns have a correlation of approximately -0.94 with interest rate shocks while we estimate this correlation as 0.05 in the data.

The rest of the paper is organized as follows: Section 2 presents the theoretical underpinnings of the generalized AELR model and shows that preferences with this specification are undefined for $\psi = 1$. We then derive the ICAPM that is consistent with $\psi \neq 1$ to show that risk premia take on large values in the neighborhood of 1. Section 3 presents our empirical analysis and findings that the real risk free rate is essentially uncorrelated with the stock market and isn't a priced state variable in the cross-section of stocks and bonds. Finally, Section 4 concludes.

2 Theory

We consider a model with shocks to consumption growth and time preferences. Thus, the model violates both of Fama (1970) assumptions. Interest rate shocks are priced relative to the CAPM and the CCAPM. The model essentially nests the long-run risk consumption growth shocks of Bansal and Yaron (2004) with the valuation shocks of AELR. The main result is that consumption growth interest rate risk has a different price than time preference interest rate risk, and the two risk premia have opposite signs when elasticity of intertemporal substitution is greater than one. Our main results are presented and discussed below; detailed derivations are in the appendix.

2.1 Setup and General Pricing Equations

Following AELR, we consider a representative agent with recursive utility function:

$$U_t = \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)} \tag{2.1}$$

where C_t is consumption at time t, δ is a positive scalar capturing time discounting, ψ is elasticity of intertemporal substitution, and $U_{t+1}^* = \left\{E_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{1/(1-\gamma)}$ is the certainty equivalent of future utility with relative risk aversion of γ. The function is defined for $\psi \neq 1$ and $\gamma \neq 1$. This utility function represents standard Epstein-Zin (EZ) preferences of Epstein and Zin (1991) and Weil (1989) except that time preferences are allowed to vary over time instead of being constant[2]. Time preferences are affected by $\frac{\lambda_{t+1}}{\lambda_t}$, which is assumed known at time t. These preferences relax the traditional restraint of recursive preferences that the aggregator function is independent of time and state. Specifically, the preferences imply dropping assumption A3 of Skiadas (2009). This utility function is not defined for $\psi = 1$. We consider alternative preferences that are defined for $\psi = 1$ at the end of this section and in the appendix.

If we restrict $\psi \neq 1$, then using standard techniques for working with EZ preferences, AELR show that equation (2.1) implies a log stochastic discount factor of:

$$m_{t+1} = \theta \log\left(\delta \frac{\lambda_{t+1}}{\lambda_t}\right) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)\, r_{w,t+1} \tag{2.2}$$

[2] Other authors have looked at allowing time preferences to vary, for example: Maurer (2012); Normandin et al. (1998)

where

$$\theta = \frac{1-\gamma}{1-1/\psi} \tag{2.3}$$

Lower case letters signify logs. Δc_{t+1} is log consumption growth from period t to period $t+1$. $r_{w,t+1}$ is the log return on the overall wealth portfolio. This stochastic discount factor is standard for EZ preferences except that time discounting (δ) is augmented by $\frac{\lambda_{t+1}}{\lambda_t}$.

We assume that innovations to consumption and expected future consumption are jointly lognormal and homoskedastic. Similarly, innovations to time preferences and expected time preferences are jointly lognormal and homoskedastic. Formally,

$$E_t\left[c_{t+a}\right] = E_{t-1}\left[c_{t+a}\right] + \varepsilon^c_{a,t} \tag{2.4}$$

$$E_t\left[\lambda_{t+1+b}\right] = E_{t-1}\left[\lambda_{t+1+b}\right] + \varepsilon^\lambda_{b,t} \tag{2.5}$$

with $\left[\left\{\varepsilon^c_{a,t}\right\}_{a>0}, \left\{\varepsilon^\lambda_{b,t}\right\}_{b>0}\right]$ distributed jointly normally with constant variance (i.e., $cov_t\left(\varepsilon^c_{a,t}, \varepsilon^\lambda_{b,t+1}\right) = V$ for all t).[3] This implies that excess returns on the wealth portfolio are lognormal and homoskedastic. For simplicity, we assume that all other excess returns are lognormal as well. Lognormality and homoscedasticity simplify the model and ensure that risk premia are constant over time, focusing attention on interest rate shocks. In their benchmark model, AELR specify a more restrictive stochastic process for λ_{t+1} and assume that expected consumption growth is constant over time. Similarly, Bansal and Yaron (2004) specify a more restrictive consumption growth process in their fluctuating growth rates model.

The stochastic discount factor of equation (2.2) can be used to price all assets. In particular, it implies a risk free rate of[4]:

$$r_{f,t+1} = -\log\left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right) + \frac{1}{\psi}E_t\left[\Delta c_{t+1}\right] - \frac{1-\theta}{2}\sigma^2_w - \frac{\theta}{2\psi^2}\sigma^2_c \tag{2.6}$$

and risk premia of:

$$E_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma^2_i = \frac{\theta}{\psi}\sigma_{ic} + (1-\theta)\,\sigma_{iw} \tag{2.7}$$

[3]Note that λ_{t+1} is known one period in advance so time t shocks to λ expectations start with λ_{t+1}.

[4]We work with real variables in our analysis though AELR also provide a version of their model that specifies an inflation process.

σ_w^2 is the variance of excess returns to the wealth portfolio. $\sigma_c^2 = var_t\left(\varepsilon_{0,t+1}^c\right)$ is consumption variance relative to expectations last period. σ_{ic} is covariance of asset i's return with current consumption shocks. σ_{iw} is covariance of asset i's return with wealth portfolio returns. $\frac{1}{2}\sigma_i^2$ is a Jensen's inequality correction for expected log returns using variance of asset i's return. From equations (2.6) and (2.7), it is clear that the real risk free interest rate changes over time in response to time preferences ($\frac{\lambda_{t+1}}{\lambda_t}$) and expected consumption growth ($E_t\left[\Delta c_{t+1}\right]$) and that risk premia are constant over time.

2.2 Substituting out Consumption (ICAPM)

Following Campbell (1993), we log-linearize the representative agent's budget constraint ($W_{t+1} = R_{w,t+1}\left(W_t - C_t\right)$) to yield:

$$r_{w,t+1} - E_t\left[r_{w,t+1}\right] = \left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j\Delta c_{t+1+j} - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j} \tag{2.8}$$

where ρ is a log-linearization constant.[5] Because risk premia are constant over time, $News_{h,t+1} \equiv \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j}$ depends solely on changes to expected interest rates, which change over time in response to time preferences and expected consumption growth as described by equation (2.6).[6] We use the identity (2.8) and the risk free rate decomposition, equation (2.6), to substitute out current consumption covariance from the risk premia in equation (2.7).

These substitutions yield the following ICAPM:

$$E_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \gamma\sigma_{iw} + \left(\gamma - 1\right)\sigma_{ih(c)} - \frac{\gamma - 1}{\psi - 1}\sigma_{ih(\lambda)} \tag{2.9}$$

Risk premia are determined by covariance with the market and covariance with state variables related to future interest rates. $\sigma_{ih(c)}$ is covariance with consumption growth shocks to future interest rates. $\sigma_{ih(\lambda)}$ is covariance with time preference shocks to future interest rates. Together,

[5] Specifically, $\rho = 1 - \exp\left(\overline{c - w}\right)$ where $\overline{c - w}$ is the average log consumption-wealth ratio. We use a monthly coefficient value of $\rho = 0.996$ in our analysis.

[6] The h subscript follows the notation of Campbell (1993) to indicate hedging of future interest rates.

they add up to covariance with overall interest rate news:

$$\begin{aligned}\sigma_{ih} &\equiv cov_t\left(r_{i,t+1}, (E_{t+1}-E_t)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j}\right) \\ &= \sigma_{ih(c)} + \sigma_{ih(\lambda)}\end{aligned} \tag{2.10}$$

The risk prices in equation (2.9) are revealing. Market return risk (σ_{iw}) is priced by relative risk aversion (γ) as in other ICAPM models. Also consistent with other ICAPM models, state variable covariance ($\sigma_{ih(c)}$ and $\sigma_{ih(\lambda)}$) is priced only if $\gamma \neq 1$. Yet, the two components of interest rate risk have different prices. Whereas $\sigma_{ih(c)}$ is priced by $\gamma - 1$, $\sigma_{ih(\lambda)}$ is priced by $-\frac{\gamma-1}{\psi-1}$. When $\psi > 1$, the prices have opposite signs, and if ψ is close to 1, time-preference risk is amplified relative to consumption growth risk. The key distinction between equation (2.9) and previous ICAPM models like Campbell (1993) is that we consider shocks to both consumption growth and time preferences. Because Campbell assumes constant preferences, he omits $\sigma_{ih(\lambda)}$ and treats σ_{ih} as equivalent to $\sigma_{ih(c)}$.

2.3 Substituting out Wealth Returns (CCAPM)

The budget constraint (equation 2.8) can also be used to substitute out covariance with wealth portfolio returns to express risk premia in terms of a generalized CCAPM along the lines of Bansal and Yaron (2004) long run risk model. The resulting pricing equation is:

$$E_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \gamma\sigma_{ic} + \left(\gamma\psi - 1\right)\sigma_{ih(c)} - \frac{\gamma\psi - 1}{\psi - 1}\sigma_{ih(\lambda)} \tag{2.11}$$

Consumption risk (σ_{ic}) is priced by relative risk aversion (γ) as in the standard CCAPM. Consistent with Bansal and Yaron (2004), interest rate risk is only priced if $\gamma \neq 1/\psi$.[7] That is, interest rate risk is priced under general EZ preferences, but not under power utility. As in our ICAPM, the most striking thing about equation (2.11) is that the two types of interest rate risk are priced differently. Once again, time preference interest rate risk differs from consumption growth interest rate risk by a factor of $\frac{-1}{\psi-1}$.

Our ICAPM, equation (2.9), and generalized CCAPM, equation (2.11), are at odds with tradi-

[7] Bansal and Yaron (2004) express their version of equation (2.11) in terms of future consumption growth. This is just a different way of describing the same relationship.

tional reasoning about interest rate risk. If one considers only consumption growth shocks, positive interest rate shocks are good news for investors under typical parameter assumptions ($\gamma > 1$ for the ICAPM and $\gamma > 1/\psi$ for the CCAPM). Thus, assets that positively covary with interest rate shocks are risky and require extra risk premia relative to CAPM and CCAPM pricing. Campbell and Viceira (2002) use this logic to argue that long term bonds are valuable hedges against interest rate decreases. If $\psi > 1$ and $\frac{1}{\psi-1}\sigma_{ih(\lambda)}$ dominates $\sigma_{ih(c)}$, the logic actually goes the opposite way: investors want to hedge against interest rate *increases*, making long term assets (including bonds) risky investments.

2.4 $\psi = 1$ and Disciplining Parameter Values

The typical strategy for working with EZ preferences to examine the $\psi = 1$ case is to take the limit of the utility function: this yields Cobb-Douglas style preferences. Specifically, if one defines the usual EZ value function[8]

$$V_t = \left[(1-\delta)C_t^{1-1/\psi} + \delta\left(V_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}$$

then taking the limit as $\psi \to 1$ can be done by L'Hopital's rule (detailed derivation in the appendix):

$$\lim_{\psi \to 1} V_t \quad = \quad C_t^{(1-\delta)}\left(V_{t+1}^*\right)^{\delta}$$

However, this procedure cannot be performed with the AELR specification because the limit diverges. Specifically

$$\lim_{\psi \to 1} lnU_t \quad = \quad \infty$$

as long as $\lambda_t + \delta \neq 1$, a condition that certainly holds as λ_t is random. A simple rescaling, on the other hand, also does not solve the problem because it creates fundamentally different preferences. Consider, for example, re-specifying the value function by dividing the taste modifier (λ_t) and the

[8]Note that the multiplication of C_t by $(1-\delta)$ is simply a rescaling in this context because $(1-\delta)$ is a constant. This is not the same thing as multiplication by random variable.

time discount factor (δ) by $\lambda_t + \delta$ in the hopes of making the above limit converge:

$$V_t = \left[\frac{\lambda_t}{\lambda_t + \delta} C_t^{1-1/\psi} + \frac{\delta}{\lambda_t + \delta} \left(V_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)} \tag{2.12}$$

In this specification, the taste shocks will accumulate. Specifically, the coefficient next to C_{t+1} will be a function of λ_t as well as λ_{t+1} which can be seen by simply iterating the value function forward one period. Consider a simple example: the agent lives for three periods, chooses consumption in period 1 and 2 and then simply consumes the remainder of wealth, W, in the third period. There is no uncertainty in this case as wealth is not random and assume all values of λ are known. Then the optimization solved by the agent is:

$$V_0 = \max_{\{C_0, C_1\}} \left[(1 - \beta_0) C_0^{1-\frac{1}{\psi}} + \beta_0 (1 - \beta_1) C_1^{1-\frac{1}{\psi}} + \beta_0 \beta_1 (1 - \beta_2) C_2^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \tag{2.13}$$

where $C_2 = W - C_0 - C_1$ and $\beta_t = \frac{\delta}{\lambda_t + \delta}$. The first order conditions for this problem imply

$$\frac{1}{\delta} \frac{\lambda_0}{\lambda_1} (\lambda_1 + \delta) \left(\frac{C_0}{C_1} \right)^{-\frac{1}{\psi}} = 1 \tag{2.14}$$

while the same setup with the AELR specification yields first order conditions that imply

$$\frac{1}{\delta} \frac{\lambda_0}{\lambda_1} \left(\frac{C_0}{C_1} \right)^{-\frac{1}{\psi}} = 1 \tag{2.15}$$

The tradeoff between consuming in the first and second period is fundamentally different in the two sets of preferences. Therefore, a simple rescaling does not solve the problem and the AELR preferences as currently specified do not admit a parameter value of $\psi = 1$. It is our view that leaving the model free to select values in this region is important in context of the aforementioned debate in the macro literature.

More generally, the insight of AELR preferences is they are able to insert a wedge between observed consumption and asset returns which have a very low correlation in the data and thus are problematic for many asset pricing models that predict a high correlation. Moreover, the state variable that breaks this correlation is observable via the risk free rate. One can take these ideas and attempt to create a preference relation that allows for all values of ψ and retains these properties.

Imagine specifying the value function as

$$V_t = \left[(1-\delta)H_t(C_t)^{1-\frac{1}{\psi}} + \delta\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \tag{2.16}$$

where H_t is a time varying function of consumption. It is simply another way to introduce the idea of taste shocks that transform the flow of utility in each period to create a wedge between asset prices and consumption. The limit of this value function as $\psi \to 1$ is well defined and is simply the Cobb-Douglas representation

$$\lim_{\psi\to 1} V_t = H_t(C_t)^{1-\delta}\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{\delta}{1-\gamma}} \tag{2.17}$$

yielding an SDF of

$$M_{t+1} = \delta \frac{V_{t+1}^{1-\gamma}}{\left(E_t V_{t+1}^{1-\gamma}\right)}\left(\frac{H_{t+1}(C_{t+1})}{H_t(C_t)}\right)^{-1}\left(\frac{H'_{t+1}(C_{t+1})}{H'_t(C_t)}\right) \tag{2.18}$$

where $H'(C) = \frac{dH(C)}{dC}$. In order for the time varying parameters in H to be observable in the risk free rate it must be the case that they appear in M_{t+1}. For example specifying $H_t(C_t) = \Lambda_t^* C_t$ would result in a model where the state variable would be entirely unobservable (this can be easily seen by the fact that Λ^* cancels in the above equation) and results in a model that has an extra degree of freedom to fit asset prices (namely, the covariance with a completely unobservable state variable). Obviously a different specification or more structure needs to be placed on $H_t(C_t)$ in order to have a model that isn't vacuous. We solve the $H_t(C_t) = \Lambda_t^* C_t$ case explicitly for a simple consumption and Λ^* process to highlight this fact. Defining $H_t(C_t) = \Lambda_t^* C_t$ implies

$$V_t = (\Lambda_t^*)^{1-\delta} C_t^{1-\delta}\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{\delta}{1-\gamma}} \tag{2.19}$$

$$M_{t+1} = \delta \frac{V_{t+1}^{1-\gamma}}{\left(E_t V_{t+1}^{1-\gamma}\right)}\left(\frac{C_{t+1}}{C_t}\right)^{-1}$$

which is the usual SDF specified in terms of the value function. To substitute out the value function, one can assume a process for log taste shock growth and log consumption growth and then guess-verify the value function. Assume (though this can be generalized using Wold's theorem to any time series process) that

$$\Delta c_{t+1} = \mu_c + \varepsilon^c_{t+1} \tag{2.20}$$

$$\Delta\lambda^*_{t+1} = \mu_{\lambda^*} + \varepsilon^{\lambda^*}_{t+1} \tag{2.21}$$

with $\varepsilon^i_t \sim N(0,\sigma^2_i)$, $\varepsilon^c_t \perp \varepsilon^{\lambda^*}_t$, $\Delta c_{t+1} \equiv \log(\frac{C_{t+1}}{C_t})$, $\Delta\lambda^*_{t+1} \equiv \log(\frac{\Lambda^*_{t+1}}{\Lambda^*_t})$. Furthermore, as in AELR, assume that Λ^*_{t+1} is known at time t. The log SDF is

$$m_{t+1} = \log\delta + (1-\gamma)v_{t+1} - \log\left(E_t V^{1-\gamma}_{t+1}\right) - \Delta c_{t+1} \tag{2.22}$$

Solving this using the method of undetermined coefficients and substituting these results into the log SDF, equation (2.22) yields the log SDF:

$$\begin{aligned} m_{t+1} &= \log\delta - \gamma\Delta c_{t+1} + \delta(1-\gamma)\Delta\lambda^*_{t+2} - (1-\gamma)(\mu_c + \delta\mu_\lambda) - \\ &- \frac{(1-\gamma)^2}{2}(\sigma^2_c + \delta^2\sigma^2_{\lambda^*}) \end{aligned} \tag{2.23}$$

The risk free rate can be derived as usual and does not depend on λ^*:

$$r_{f,t+1} = -\log\delta + \mu_c + \frac{(1-2\gamma)}{2}\sigma^2_c \tag{2.24}$$

On the other hand, the risk premia do depend on λ^* :

$$E_t r_{i,t+1} + \frac{1}{2}\sigma^2_i - r_{f,t+1} = \gamma Cov(r_{i,t+1}, \Delta c_{t+1}) + \delta(\gamma-1)Cov(r_{i,t+1}, \Delta\lambda^*_{t+2}) \tag{2.25}$$

Therefore, expected excess asset returns are a function of log consumption growth as well as log taste growth. In this model, λ^* serves as a free parameter because it is never observed but influences risk premia. In other words, what matters for risk premia is λ^* while λ is what matters for the risk free rate.

While the case of $\psi = 1$ is not defined with the AELR specification, we can introspect about what are reasonable values for ψ and γ along the lines of Epstein et al. (2014). To generate better intuition for how close ψ can be to 1, we propose a thought experiment with simple consumption and time preference processes. Specifically, consider a three period economy with constant perishable consumption endowments of $C_0 = C_1 = C_2 = C$ in each period. Time preferences are known in advance for periods 0 and 1. For simplicity we assume $\lambda_0 = \lambda_1 = 1$ and we also assume $\delta = 1$. The only uncertainty in the economy is period 2 time preferences, which are revealed at time 1. λ_2 takes on two possible values, λ_H or λ_L with probabilities π_H and π_L, respectively. We want to know how the representative agent values wealth in state L relative to state H.

In the appendix, we derive Arrow-Debreu state prices for the two states and find that their ratio is:

$$\frac{P_L}{P_H} = \frac{\pi_L}{\pi_H}\left(\frac{1+\lambda_L}{1+\lambda_H}\right)^{-\frac{\gamma-1/\psi}{1-1/\psi}} \tag{2.26}$$

Note that these are prices at time 0 for state-contingent payoffs at time 1. Under power utility with $\gamma = 1/\psi$, the price ratio is simply the probability ratio. This is exactly what we should expect. With power utility, marginal utility of wealth is pinned down by consumption and current time preferences, which is constant across states. By contrast, state prices are highly sensitive to future time preferences when $1/\psi$ differs from γ and is close to 1. We do not have great intuition for whether $-\frac{\gamma-1/\psi}{1-1/\psi}$ should be positive or negative, but we believe its magnitude should be small.

To be more concrete, assume $\pi_L = \pi_H = 0.5$, $\lambda_H = 1$, and $\lambda_L = 0.9$. Table 1 presents the equation (2.26) state price ratio for these parameters at various values of γ and ψ. Parameterizations with $\gamma > 1$ are in Panel A. Parameterizations with $\gamma < 1$ are in Panel B. The upward sloping diagonals of 1's in both panels represent power utility with $\gamma = 1/\psi$.

What are reasonable values for $\frac{P_L}{P_H}$? The thought experiment is what you would pay for an extra dollar in a state in which time preferences will soon fall versus an extra dollar in a state in which time preferences will remain constant, keeping in mind that current and future consumption are the same in both states. As a starting point, we propose that it is difficult to rationalize state price ratios larger in magnitude than the ratio of the time preference shock itself. In Table 1, ratios between 0.95 and 1.05 are in bold italics, and ratios between 0.9 and 1.1 are highlighted in italics.[9]

[9]The broader range requires that $\frac{P_L \pi_H}{P_H \pi_L}$ falls between $\left(\frac{\lambda_L}{\lambda_H}\right)$ and $\left(\frac{\lambda_L}{\lambda_H}\right)^{-1}$. The narrower range requires that $\frac{P_L \pi_H}{P_H \pi_L}$ falls between $\left(\frac{1+\lambda_L}{1+\lambda_H}\right)$ and $\left(\frac{1+\lambda_L}{1+\lambda_H}\right)^{-1}$, which is equivalent to the condition that $|\gamma - 1/\psi| \leq |1 - 1/\psi|$.

As expected, ratios in these ranges require $1/\psi$ to be close to γ or far from 1. For example, if γ is 5, ψ must be less than 0.44. With lower relative risk aversion, ψ can be closer to one without posing a problem.

3 Empirical Analysis

Our empirical focus is not to test the model discussed in the previous section but rather to directly address the question of whether real interest rate risk is priced. This question is actually a bit at odds with the model in that it implies a single type of interest rate risk whereas the model shows that their are two different interest rate factors with different risk prices. Ideally, we would like to separately measure consumption growth and time preference interest rate risk. Given the unobservability of time preferences and the imprecise and low-frequency nature of consumption data, measuring aggregate interest rate risk is probably the best we can do. Moreover, aggregate interest rate risk is of direct interest because interest rates are highly visible and economically important. Even though we don't directly test it, the model does inform how we think about and measure interest rate risk. Perhaps most significantly, the model predicts that investors care about shocks to both current and expected future risk free interest rates. Thus, instead of considering just $cov_t\left(r_{i,t+1}, r_{f,t+2} - E_t\left[r_{f,t+2}\right]\right)$, we focus on $\sigma_{ih} \equiv cov_t\left(r_{i,t+1}, \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}\right)$.

Our empirical work faces two primary challenges. First, our focus is on real interest rates. This is the risk free rate in our model, and it is the relevant quantity for actual economic decisions. Unfortunately, real interest rates are not directly observed. We overcome this problem by modeling expected Consumer Price Index (CPI) inflation and estimating monthly real interest rates as the difference between nominal 1-month Treasury bill interest rates and expected inflation over the next month. For our baseline estimates, we focus on the 1983 to 2012 time period because monetary policy has been more consistent and inflation has been less volatile during the Greenspan and Bernanke Federal Reserve chairmanships than in previous periods.

Our second empirical challenge is that interest rate risk involves shocks to expectations. Thus, we need to estimate interest rate expectations. We do this with a vector autoregression (VAR) of interest rates, inflation, and other state variables. From the VAR, we extract an estimate for the time series of $\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}$ innovations, which we in turn use to estimate σ_{ih} for various assets.

3.1 Vector Autoregression

Our VAR model is:

$$Y_t = AY_{t-1} + \omega_t \tag{3.1}$$

Y_t is a $k \times 1$ vector with the nominal 1-month treasury bill log yield and seasonally adjusted log CPI inflation over the past month as its first two elements. The remaining elements of Y_t are state variables useful for forecasting these two variables. The assumption that the VAR model has only one lag is not restrictive because lagged variables can be included in Y_t. We demean Y_t before estimating the VAR to avoid the need for a constant in equation (3.1).

We define vector ei to be the ith column of a $k \times k$ identity matrix. Using this notation we can extract expectations and shocks to current and future expectations from Y_t, A, and ω_t. Our interest is in the real risk free interest rate, which we estimate as the nominal 1-month treasury bill yield less expected inflation:

$$r\widehat{_{f,t+1}} = (e1' - e2'A)\, Y_t \tag{3.2}$$

Similarly, expected future risk free rates are:

$$E_t\left[r\widehat{_{f,t+j}}\right] = (e1' - e2'A)\, A^{j-1} Y_t \tag{3.3}$$

Shocks to current and expected risk free rates are:

$$(E_{t+1} - E_t)\, r_{f,\widehat{t+1}+j} = (e1' - e2'A)\, A^{j-1} \omega_{t+1} \tag{3.4}$$

Most importantly, total interest rate news is:

$$\begin{aligned} News_{h,t+1} &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,\widehat{t+1}+j} \\ &= (e1' - e2'A) \sum_{j=1}^{\infty} \rho^j A^{j-1} \omega_{t+1} \\ &= (e1' - e2'A)\, \rho\, (I - \rho A)^{-1}\, \omega_{t+1} \end{aligned} \tag{3.5}$$

where I is the identity matrix.

All that remains is to choose state variables for Y_t and estimate equation (3.1). Following Campbell (1996), we include the relative treasury bill rate, defined as the difference between the current one-month treasury bill yield and the average one-month treasury bill yield over the previous 12 months. Similarly, we include the relative monthly CPI inflation rate, defined the same way. Next, we include the yield spread between 10-year treasury bonds and 3-month treasury bonds because the slope of the yield curve is known to predict interest rate changes. Finally, we include the CRSP value-weighted market return and the log dividend-price ratio (defined as dividends over the past year divided by current price), which is known to predict market returns. These variables are useful to the extent that equity returns are related to expected future interest rates. We considered including additional lags of these variables by re-estimating equation (3.1) with multiple lags of Y_t. The Bayesian Information Criteria is insensitive to adding lags so we do not include lagged variables in Y_t.

Table 2 shows coefficient estimates and standard errors for the elements of A related to predicting nominal interest rates and inflation. Columns (1) and (2) report results for the 1983 to 2012 time period, which is our primary focus. Nominal interest rate shocks are highly persistent with lag coefficient of 0.96. Inflation shocks are much less persistent and only have a lag coefficient of 0.07. Inflation is increasing in lagged nominal yields. The VAR explains 95% of the variation in nominal yields over time. Inflation changes are less predictable with an R-squared of 0.24.

Because our main interest is in the risk free rate, we plot $\widehat{r_{f,t+1}}$ in Figure 1. Along with our estimated real risk free rate, we also plot the nominal one-month treasury bill yield and the Federal Reserve Bank of Cleveland's real risk free rate estimate.[10] As we would expect in a stable inflation environment, real interest rates generally follow the same pattern as nominal interest rates. Nonetheless, inflation expectations do change over time, particularly over the past few years. Our real risk free rate estimate closely tracks the Federal Reserve Bank of Cleveland's estimate, which increases our confidence in our methodology.

As a robustness check, we also estimate real risk free rates and real risk free rate news over a longer time period, starting in 1927. Our methodology for the longer time period is the same as

[10] The Federal Reserve Bank of Cleveland's real risk free rate estimates are described by Haubrich et al. (2008, 2012).

before except that we use the unadjusted CPI because the seasonally adjusted CPI is only available starting in 1947. Columns (3) and (4) of Table 2 report the VAR results. In the extended time sample, inflation shocks are more persistent (inflation's lagged coefficient is 0.78, compared to 0.07 before). The results are otherwise similar to the original VAR. Figure 2 plots nominal and estimated real interest rates from 1927 to 2012. Expected inflation varies more in the extended sample than it does after 1983. Thus, the real and nominal interest rates do not track each other as closely. Expected inflation is particularly high in the 1930's, 1940's, and 1970's, and deflation caused real interest rates to exceed nominal interest rates in the 1920's.

3.2 Cross-Sectional Equity Pricing

If real interest rate risk is priced and stocks vary in their exposure to real interest rate risk, real interest rate risk should be priced in the cross section of stock returns. This is not the first paper to connect time series interest rate changes with cross-sectional stock returns. For example, Fama and French (1993) find comovement between excess stock returns and excess returns on long term bonds but conclude that bond factors have little impact on cross sectional stock prices. Petkova (2006) finds that innovations to term spreads and one month nominal interest rates are correlated with and partially explain size and value returns. Nieuwerburgh et al. (2012) find that high returns to value stocks relative to growth stocks are explained by covariance with shocks to nominal bond risk premia whereas returns to treasury bond portfolios of different maturities are largely explained by differential exposure to the level of interest rates. Our empirical analysis differs from previous studies because we focus specifically on stock exposure to real interest rate innovations. Moreover, we sort stocks based on this exposure instead of focusing on established size and value returns.

To test whether interest rate risk is priced we sort stocks into portfolios according to covariance with interest rate news ($News_{h,t+1}$). Specifically, we estimate $\sigma_{ih} = cov_t\left(r_{i,t+1}, News_{h,t+1}\right)$ on a rolling basis for all NYSE, AMEX, and NASDAQ common stocks using returns and VAR $News_h$ estimates over the past three years, with the requirement that included stocks must have at least two years of historical data. Value-weighted decile portfolios are formed monthly by sorting stocks according to those estimates.

Table 3 reports market capitalization, average excess returns, and $\beta_{ih} = \frac{\sigma_{ih}}{\sigma_h^2}$ estimates for each portfolio. The table also reports pricing errors (alphas) relative to the CAPM and Fama and

French (1993) three factor model and factor loadings (betas) for the three factor model. Panel A reports results for our baseline 1985-2012 time period.[11] Risk free rate news betas increase across the portfolios, and decile 10's news beta is a significant 0.58 higher than decile 1's news beta. Monthly excess returns are 42 bps lower in the 10th decile than in the 1st decile, but this return difference is not statistically significant, and there is no clear pattern to excess returns across the decile portfolios other than a drop in returns in decile 10. CAPM and 3 Factor alphas follow the same basic pattern. Factor loadings are also similar across the portfolios. The one exception is that decile 10 has a large negative loading on the value factor (HML). The bottom line is that there is no evidence that interest rate risk is priced in the cross section of equities.

Results are similar in the extended 1929-2012 sample, reported in Panel B. Once again, average excess returns and alpha estimates decrease with interest rate news exposure, but the differences are not significant. The most striking difference between Panel A and Panel B is that β_{ih} differences across the portfolios are not significant in the extended sample. This suggests that stock-level interest rate risk was not stable over time early in the sample, undercutting our ability to form interest rate risk portfolios. This problem appears to be concentrated in the first few decades of the sample when inflation and interest rates were most volatile. In later analysis, we examine a 1952 to 2012 sample and find significant β_{ih} differences between the decile portfolios. As in the other samples, these β_{ih} differences are not accompanied by significant return differences.

3.3 Equity Premium

Because the market portfolio is a claim to future dividends, it may be exposed to interest rate risk. Thus, interest rate risk may affect expected equity returns and could explain part of the equity premium puzzle. The magnitude and direction of this effect depend on the market return's covariance with interest rate news and the price of interest rate risk.

AELR imply that interest rate risk explains virtually all of the equity premium. In their benchmark model, assets are priced based on covariance with consumption growth shocks and time preference shocks, which map directly into interest rate shocks. Consistent with previous studies, they estimate that equity returns are essentially uncorrelated with consumption growth. Thus,

[11] We form the portfolios based on at least two years of historical data, which causes the sample to start in 1985 instead of 1983.

their explanation of the equity premium is almost entirely based on interest rate risk. Equities are risky because they have a long duration and are sensitive to persistent real interest rate shocks. Duration simultaneously explains the upward sloping yield curve and the equity premium. In the AELR benchmark model, equity returns are highly sensitive to interest rate shocks, with a correlation of approximately -0.94.

Using our estimates of interest rate news, we can directly measure these two moments. Panel A of Table 4 shows results for the 1985 to 2012 time period. Excess market returns ($rmrf$) have a correlation of 0.05 and a beta of 0.11 with respect to interest rate news. These estimates are close to zero, suggesting that equity returns have little exposure to interest rate risk. According to the point estimate, the market return is positively correlated with interest rate shocks, consistent with long run consumption growth shocks and in contrast to AELR's time preference shocks.

Table 4 also reports interest rate correlations and betas for the long-short decile 10 minus decile 1 interest rate risk portfolio and for 1 to 2 year and 5 to 10 year bonds.[12] By construction, the long-short interest rate risk portfolio has a positive beta. The bond portfolios have negative exposures to interest rate news. However, these exposures are small. Interest rate betas are -0.04 for both portfolios, and the beta is only significantly different from zero for the short-term bonds.

The final rows of Table 4 report average excess returns and average excess returns divided by interest rate news beta. If interest rate news is the primary risk factor investors care about, this ratio (the implied price of beta) should be consistent across assets. The point estimates clearly differ. In particular, the bond returns and cross-sectional interest rate risk portfolio imply a negative price of interest rate risk whereas market returns imply a positive price. Unfortunately, betas and average returns are measured too imprecisely to definitively rule out consistent interest rate risk pricing across the assets. Panel B of Table 4 reports the same statistics for a longer sample period, starting in 1952 when CRSP bond return data starts. The basic results are all the same.

Our findings suggest that interest rate risk is unlikely to explain the equity premium. Certainly, there is no evidence in favor of the hypothesis that equities face significant interest rate risk. How can this be reconciled with AELR's empirical findings? The main difference between our analysis and AELR's is that AELR do not estimate real interest rate innovations. Their GMM includes the unconditional correlation between equity returns and the real risk free rate at an annual frequency

[12] Bond return data is from CRSP.

but omits the more important correlation of interest rate *news* with *excess* equity returns. Our analysis estimates this moment and finds that it is essentially zero.

4 Conclusion

Is real interest rate risk priced? Theoretically, it could be priced in either direction. Empirically, there is little evidence that real interest rate risk is priced at all.

Our interest rate risk model has two theoretical implications. First, it matters where interest rate shocks comes from. Interest rate increases stemming from news about future consumption growth are generally good news to investors whereas interest rate increases stemming from time preference shocks are generally bad news. Thus, long-run consumption risk logic implies that long-duration assets are relatively safe whereas time preference risk logic implies that long-duration assets are relatively risky. A more general lesson is the importance of thinking in general equilibrium terms. Because interest rates are endogenous, interest rate risk is not a meaningful concept without specifying what is driving interest rate shocks.

The second theoretical implication of our model is that AELR preferences with ψ close to 1 and significantly different from $1/\gamma$ imply implausible aversion to future time preference shocks.

Empirically, stocks sorted on interest rate risk have only small, statistically insignificant return differences. Moreover, the market return and treasury bond returns have low covariance with interest rate news. Thus, interest rate risk is unlikely to explain much of equity or bond return premia even if it is priced to some extent in the cross section. Overall, our results suggest that interest rate risk is not a major concern to investors.

A Appendix

A.1 Setup and General Pricing Equations

A.1.1 $\psi \neq 1$

The representative agent has the augmented Epstein-Zin preferences described by equation (2.1):

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}$$

where $U_{t+1}^* = \left\{E_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{1/(1-\gamma)}$ is the certainty equivalent of future utility. Optimization is subject to budget constraint:

$$W_{t+1} = R_{w,t+1}\left(W_t - C_t\right) \tag{A.1}$$

where W_t is wealth at time t and $R_{w,t+1}$ is the return on the overall wealth portfolio, which is a claim to all future consumption.

AELR use standard techniques from the Epstein-Zin preference literature to show that the preferences represented by equation (2.1) imply the log stochastic discount factor (sdf):

$$m_{t+1} = \theta \log\left(\delta \frac{\lambda_{t+1}}{\lambda_t}\right) - \frac{\theta}{\psi}\Delta c_{t+1} + \left(\theta - 1\right) r_{w,t+1}$$

This sdf should not be surprising. It is just the standard Epstein-Zin sdf with time-varying time discounting (i.e., $\delta \frac{\lambda_{t+1}}{\lambda_t}$ instead of δ).

Using $0 = E_t\left[m_{t+1} + r_{i,t+1}\right] + \frac{1}{2}\left(\sigma_m^2 + \sigma_i^2 + 2\sigma_{mi}\right)$ (the log version of $1 = E_t\left[M_{t+1} R_{i,t+1}\right]$), we calculate the expected return for any asset as:

$$\begin{aligned} E_t\left[r_{i,t+1}\right] + \frac{1}{2}\sigma_i^2 &= -\theta \log\left(\delta \frac{\lambda_{t+1}}{\lambda_t}\right) + \frac{\theta}{\psi} E_t\left[\Delta c_{t+1}\right] + \left(1-\theta\right) E_t\left[r_{w,t+1}\right] \\ &\quad -\frac{1}{2}\left(\frac{\theta}{\psi}\right)^2 \sigma_c^2 - \frac{1}{2}\left(1-\theta\right)^2 \sigma_w^2 + \frac{\theta}{\psi}\left(\theta - 1\right)\sigma_{wc} \\ &\quad + \frac{\theta}{\psi}\sigma_{ic} + \left(1-\theta\right)\sigma_{iw} \end{aligned} \tag{A.2}$$

The $\frac{1}{2}\sigma_i^2$ on the left hand side of equation (A.2) is a Jensen's inequality correction for log returns.

The risk free rate is of particular interest:

$$\begin{aligned} r_{f,t+1} \quad &= \quad -\theta \log \left(\delta \frac{\lambda_{t+1}}{\lambda_t} \right) + \frac{\theta}{\psi} E_t \left[\Delta c_{t+1} \right] + (1-\theta) E_t \left[r_{w,t+1} \right] \\ & \quad -\frac{1}{2} \left(\frac{\theta}{\psi} \right)^2 \sigma_c^2 - \frac{1}{2} (1-\theta)^2 \sigma_w^2 + \frac{\theta}{\psi} (\theta - 1) \sigma_{wc} \end{aligned} \tag{A.3}$$

Differencing equations (A.2) and (A.3) yields the risk premia of equation (2.7):

$$E_t \left[r_{i,t+1} \right] - r_{f,t+1} + \frac{1}{2} \sigma_i^2 = \frac{\theta}{\psi} \sigma_{ic} + (1-\theta) \sigma_{iw}$$

which is exactly the same expression as in standard Epstein-Zin models. Substituting $E_t \left[r_{w,t+1} \right]$ into equation (A.3), yields equation (2.6):

$$r_{f,t+1} = -\log \left(\delta \frac{\lambda_{t+1}}{\lambda_t} \right) + \frac{1}{\psi} E_t \left[\Delta c_{t+1} \right] - \frac{1-\theta}{2} \sigma_w^2 - \frac{\theta}{2\psi^2} \sigma_c^2$$

which is the same as standard Epstein-Zin models except that δ is replaced by $\delta \frac{\lambda_{t+1}}{\lambda_t}$.

A.1.2 $\psi = 1$

The limit of the value function as $\psi \to 1$ under AELR preferences does not exist. In the case of vanilla EZ preferences, one can find the limit of the value function by using L'Hopital's rule:

$$V_t = \left[(1-\delta) C_t^{1-1/\psi} + \delta \left(V_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

$$\begin{aligned} lnV_t \quad &= \quad \frac{1}{1-\frac{1}{\psi}} ln \left[(1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left(V_{t+1}^* \right)^{1-\frac{1}{\psi}} \right] \\ \lim_{\psi \to 1} lnV_t \quad &= \lim_{\psi \to 1} \quad \frac{\frac{1}{(1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left(V_{t+1}^* \right)^{1-\frac{1}{\psi}}} \left(\frac{1}{\psi^2} (1-\delta) C_t^{1-\frac{1}{\psi}} lnC_t + \frac{1}{\psi^2} \delta \left(V_{t+1}^* \right)^{1-\frac{1}{\psi}} ln \left(V_{t+1}^* \right) \right)}{\frac{1}{\psi^2}} \\ &= \quad (1-\delta) lnC_t + \delta ln \left(V_{t+1}^* \right) \\ V_t \quad &= \quad C_t^{(1-\delta)} \left(V_{t+1}^* \right)^{\delta} \end{aligned}$$

However, this procedure cannot be performed with the AELR specification because the limit diverges. Specifically

$$\begin{aligned}\lim_{\psi\to 1} lnU_t &= \lim_{\psi\to 1} \frac{ln\left[\lambda_t C_t^{1-1/\psi} + \delta\left(U_{t+1}^*\right)^{1-1/\psi}\right]}{1-1/\psi} \\ &= \frac{ln(\lambda_t+\delta)}{0} \to \infty\end{aligned}$$

We also examine alternative preferences that use a generalized form of consumption in EZ preferences that are defined for all ψ. Specifying the value function as

$$V_t = \left[(1-\delta)H_t(C_t)^{1-\frac{1}{\psi}} + \delta\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

Defining $H_t(C_t) = \Lambda_t^* C_t$ implies

$$V_t = (\Lambda_t^*)^{1-\delta} C_t^{1-\delta}\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{\delta}{1-\gamma}}$$

$$\begin{aligned}M_{t+1} &= \delta\frac{V_{t+1}^{1-\gamma}}{\left(E_t V_{t+1}^{1-\gamma}\right)}\left(\frac{\Lambda_t^* C_t}{\Lambda_{t+1}^* C_{t+1}}\right)\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right) \\ &= \delta\frac{V_{t+1}^{1-\gamma}}{\left(E_t V_{t+1}^{1-\gamma}\right)}\left(\frac{C_{t+1}}{C_t}\right)^{-1} \qquad (A.4)\end{aligned}$$

which is the usual SDF specified in terms of the value function. To substitute out the value function, one can assume a process for log taste shock growth and log consumption growth and then guess-verify the value function. Assume:

$$\Delta c_{t+1} = \mu_c + \varepsilon_{t+1}^c \qquad (A.5)$$

$$\Delta\lambda_{t+1}^* = \mu_{\lambda^*} + \varepsilon_{t+1}^{\lambda^*} \qquad (A.6)$$

with $\varepsilon_t^i \sim N(0, \sigma_i^2)$, $\varepsilon_t^c \perp \varepsilon_t^{\lambda^*}$, $\Delta c_{t+1} \equiv \log(\frac{C_{t+1}}{C_t})$, $\Delta\lambda^*_{t+1} \equiv \log(\frac{\Lambda^*_{t+1}}{\Lambda^*_t})$, and that Λ^*_{t+1} is known at time t. The log SDF is

$$m_{t+1} = \log\delta + (1-\gamma)v_{t+1} - \log\left(E_t V_{t+1}^{1-\gamma}\right) - \Delta c_{t+1} \tag{A.7}$$

Guess that the log value function is

$$v_t = A_0 + A_1 c_t + A_2\lambda^*_t + A_3\lambda^*_{t+1} \tag{A.8}$$

Then

$$\begin{aligned}
\log\left(E_t V_{t+1}^{1-\gamma}\right) &= \log E_t\left[\exp\left\{(1-\gamma)(A_0 + A_1 c_{t+1} + A_2\lambda^*_{t+1} + A_3\lambda^*_{t+2}\right\}\right] \\
&= \log\left[\exp\left\{(1-\gamma)(A_0 + A_1 c_t + (A_2 + A_3)\lambda^*_{t+1})\right\} E_t\left(\exp\left\{(1-\gamma)(A_1\Delta c_{t+1} + A_3\Delta\lambda^*_{t+2})\right\}\right)\right] \\
&= (1-\gamma)(A_0 + (A_2 + A_3)\lambda^*_{t+1} + A_1 c_t + A_1\mu_c + A_3\mu_{\lambda^*}) + \frac{(1-\gamma)^2}{2}\left[A_1^2\sigma_c^2 + A_3^2\sigma_{\lambda^*}^2\right] \qquad \text{(A.9)}
\end{aligned}$$

Using the usual method of undetermined coefficients:

$$\begin{aligned}
A_0 + A_1 c_t + A_2\lambda^*_t + A_3\lambda^*_{t+1} &= (1-\delta)\lambda^*_t + (1-\delta)c_t \\
&+\frac{\delta}{1-\gamma}\left[(1-\gamma)(A_0 + (A_2 + A_3)\lambda^*_{t+1} + A_1 c_t + A_1\mu_c + A_3\mu_{\lambda^*})\right] \\
&+\frac{\delta}{1-\gamma}\left[\frac{(1-\gamma)^2}{2}\left[A_1^2\sigma_c^2 + A_3^2\sigma_{\lambda^*}^2\right]\right]
\end{aligned}$$

$$A_1 = 1 \tag{A.10}$$

$$A_2 = (1-\delta) \tag{A.11}$$

$$A_3 = \delta \tag{A.12}$$

$$A_0 = \frac{\delta}{1-\delta}(\mu_c + \mu_{\lambda^*} + \frac{(1-\gamma)}{2}(\sigma_c^2 + \delta^2\sigma_{\lambda^*}^2)) \tag{A.13}$$

Substituting these results into the log SDF yields

$$m_{t+1} = \log\delta + (1-\gamma)(A_0 + c_{t+1} + \lambda^*_{t+1} + \delta\Delta\lambda^*_{t+2})$$

$$- \left[(1-\gamma)(A_0 + \lambda^*_{t+1} + c_t + \mu_c + \delta\mu_{\lambda^*}) + \frac{(1-\gamma)^2}{2}\left[\sigma_c^2 + \delta^2\sigma^2_{\lambda^*}\right] \right] - \Delta c_{t+1} \quad \text{(A.14)}$$

$$= \log\delta - \gamma\Delta c_{t+1} + \delta(1-\gamma)\Delta\lambda^*_{t+2} - (1-\gamma)(\mu_c + \delta\mu_{\lambda^*}) - \frac{(1-\gamma)^2}{2}(\sigma_c^2 + \delta^2\sigma^2_{\lambda^*}) \quad \text{(A.15)}$$

A.2 Substituting out Consumption (ICAPM)

Following Campbell (1993) we log linearize the budget constraint to yield equation (2.8):

$$r_{w,t+1} - E_t\left[r_{w,t+1}\right] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta c_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j}$$

where $\rho = 1 - \exp\left(\overline{c-w}\right)$ is a log-linearization constant ($\overline{c-w}$ is the average log consumption-wealth ratio). Rearranging, we can express current consumption shocks as:

$$\begin{aligned} \Delta c_{t+1} - E_t\left[\Delta c_{t+1}\right] &= r_{w,t+1} - E_t\left[r_{w,t+1}\right] \\ &\quad + (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j} \\ &\quad - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \Delta c_{t+1+j} \end{aligned} \quad \text{(A.16)}$$

So far, we have only made use of modified Epstein-Zin preferences and the budget constraint. We now use assumptions about consumption and time preference innovations for the first time. Due to our homoscedasticity assumption, risk premia do not change over time, and the risk free rate only changes in response to time preference and consumption growth innovations. Thus, innovations to expected returns can be decomposed as:

$$\begin{aligned} (E_{t+1} - E_t)\, r_{w,t+1+j} &= (E_{t+1} - E_t)\, r_{f,t+1+j} \\ &= (E_{t+1} - E_t)\log\left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}}\right) + \frac{1}{\psi}(E_{t+1} - E_t)\left[\Delta c_{t+j+1}\right] \end{aligned} \quad \text{(A.17)}$$

for $j \geq 1$. Substituting equation (A.17) into equation (A.16) yields:

$$\begin{aligned} \Delta c_{t+1} - E_t\left[\Delta c_{t+1}\right] &= r_{w,t+1} - E_t\left[r_{w,t+1}\right] \\ &\quad - \left(1 - \frac{1}{\psi}\right)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \Delta c_{t+1+j} \end{aligned}$$

$$+ (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right) \tag{A.18}$$

Substituting out consumption shock covariance (σ_{ic}) from equation (2.7) yields risk premia as a function of covariances with market returns and innovations to future time preferences and consumption growth:

$$\begin{aligned} E_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 \quad = \quad & \gamma \sigma_{iw} \\ & + (\gamma - 1) \frac{1}{\psi} cov_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \right) \\ & + \frac{\theta}{\psi} cov_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right) \right) \end{aligned} \tag{A.19}$$

Equation (2.9) expresses this as:

$$E_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \gamma \sigma_{iw} - \frac{\gamma - 1}{\psi - 1} \sigma_{ih(\lambda)} + (\gamma - 1) \sigma_{ih(c)}$$

where

$$\sigma_{ih(\lambda)} = cov_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right) \right) \tag{A.20}$$

and

$$\sigma_{ih(c)} = \frac{1}{\psi} cov_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \right) \tag{A.21}$$

are the two different types of interest rate news covariance.

Another way to see this is to change notation to consider time preference shocks in the same units as consumption. Specifically, consider augmented consumption, defined as:

$$\widetilde{C}_t \equiv \lambda_t^* C_t \tag{A.22}$$

where

$$\lambda_t^* \equiv \lambda_t^{1/(1-1/\psi)} \tag{A.23}$$

With this notation change, equation (2.1) is transformed into standard Epstein-Zin preferences with respect to augmented consumption. All of Campbell (1993) and Bansal and Yaron (2004) results hold with respect to augmented consumption and returns measured in units of augmented consumption. In particular, the augmented risk free rate is:

$$\widetilde{r}_{f,t+1} = -\log(\delta) + \frac{1}{\psi} E_t [\Delta \widetilde{c}_{t+1}] - \frac{1-\theta}{2} \sigma_w^2 - \frac{\theta}{2\psi^2} \sigma_c^2 \tag{A.24}$$

and the risk premium for any asset is given by

$$E_t [\widetilde{r}_{i,t+1}] - \widetilde{r}_{f,t+1} + \frac{1}{2} \sigma_i^2 = \gamma \sigma_{iw} + (\gamma - 1) \sigma_{ih(\widetilde{c})} \tag{A.25}$$

where tildes represent augmented consumption and returns. Using the identities $\widetilde{r}_{i,t+1} = r_{i,t+1} + \frac{1}{1-1/\psi} \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$ and $\Delta \widetilde{c}_{t+1} = \Delta c_{t+1} + \frac{1}{1-1/\psi} \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$, equations (A.24) and (A.25) are equivalent to equations (2.6) and (2.9). The time preference risk premia in equations (2.9) and (2.11) blow up as ψ gets close to 1 because time preferences (λ_t) have an outsized impact on augmented consumption through $\lambda_t^* = \lambda_t^{1/(1-1/\psi)}$.

A.3 Substituting out Wealth Returns (CCAPM)

We can also use the budget constraint to substitute out wealth portfolio return covariance (σ_{iw}) from equation (2.7) by rearranging equation (A.18) and using it to decompose σ_{iw}, thereby yielding equation (2.11):

$$E_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2} \sigma_i^2 = \gamma \sigma_{ic} + (\gamma\psi - 1) \sigma_{ih(c)} - \frac{\gamma\psi - 1}{\psi - 1} \sigma_{ih(\lambda)}$$

A.4 Disciplining Parameter Values

In a three period setting with $\lambda_0 = \lambda_1 = \delta = 1$, AELR utility can be expressed as:

$$U_0 = \max_{C_0} \left\{ C_0^{1-1/\psi} + \left(E_0 \left[\max_{C_1, C_2} \left\{ C_1^{1-1/\psi} + \lambda_2 C_2^{1-1/\psi} \right\}^{\frac{1-\gamma}{1-1/\psi}} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{1/(1-1/\psi)} \tag{A.26}$$

The Euler equation for an Arrow-Debreu security that pays off in state s is:

$$P_s C_0^{-1/\psi} = \left[\pi_L \left(C_1^{1-1/\psi} + \lambda_L C_2^{1-1/\psi} \right)^{\frac{1-\gamma}{1-1/\psi}} + \pi_H \left(C_1^{1-1/\psi} + \lambda_H C_2^{1-1/\psi} \right)^{\frac{1-\gamma}{1-1/\psi}} \right]^{\frac{\gamma-1/\psi}{1-\gamma}}$$

$$*\pi_s \left(C_1^{1-1/\psi} + \lambda_L C_2^{1-1/\psi}\right)^{\frac{1/\psi-\gamma}{1-1/\psi}} * C_1^{-1/\psi} \tag{A.27}$$

where P_s is the state price for state s, π_s is the probability of state s, and λ_s is the value of λ_2 in state s.

Under our assumption that $C_0 = C_1 = C_2 = C$, equation (A.27) reduces to:

$$P_s = \pi_s \left(1 + \lambda_s\right)^{\frac{1/\psi-\gamma}{1-1/\psi}} \left[\pi_L \left(1 + \lambda_L\right)^{\frac{1-\gamma}{1-1/\psi}} + \pi_H \left(1 + \lambda_H\right)^{\frac{1-\gamma}{1-1/\psi}}\right]^{\frac{\gamma-1/\psi}{1-\gamma}} \tag{A.28}$$

Equation (A.28) immediately implies the state price ratio given by equation (2.26):

$$\frac{P_L}{P_H} = \frac{\pi_L}{\pi_H} \left(\frac{1+\lambda_L}{1+\lambda_H}\right)^{-\frac{\gamma-1/\psi}{1-1/\psi}}$$

References

Albuquerque, R., Eichenbaum, M., Luo, V. and Rebelo, S. (2016), 'Valuation risk and asset pricing', *Journal of Finance* .

Bansal, R. and Yaron, A. (2004), 'Risks for the long run: A potential resolution of asset pricing puzzles', *The Journal of Finance* **59**(4), 1481–1509.

Campbell, J. Y. (1993), 'Intertemporal asset pricing without consumption data', *The American Economic Review* pp. 487–512.

Campbell, J. Y. (1996), 'Understanding risk and return', *Journal of Political Economy* pp. 298–345.

Campbell, J. Y. and Viceira, L. M. (2002), *Strategic asset allocation: portfolio choice for long-term investors*, Oxford University Press, USA.

Epstein, L. G., Farhi, E. and Strzalecki, T. (2014), 'How much would you pay to resolve long-run risk?', *The American Economic Review* **104**(9), 2680–2697.

Epstein, L. G. and Zin, S. E. (1991), 'Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis', *Journal of political Economy* pp. 263–286.

Fama, E. F. (1970), 'Multiperiod consumption-investment decisions', *The American Economic Review* **60**(1), 163–174.

Fama, E. F. and French, K. R. (1993), 'Common risk factors in the returns on stocks and bonds', *Journal of Financial Economics* **33**(1), 3–56.

Guvenen, F. (2006), 'Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective', *Journal of Monetary Economics* **53**(7), 1451–1472.

Hall, R. E. (1988), 'Intertemporal substitution in consumption', *Journal of Political Economy* **96**(2), 339–357.

Haubrich, J. G., Pennacchi, G. and Ritchken, P. H. (2008), 'Estimating real and nominal term structures using treasury yields, inflation, inflation forecasts, and inflation swap rates', *Working Paper* .

Haubrich, J., Pennacchi, G. and Ritchken, P. (2012), 'Inflation expectations, real rates, and risk premia: evidence from inflation swaps', *Review of Financial Studies* **25**(5), 1588–1629.

Maurer, T. (2012), 'Is consumption growth merely a sideshow in asset pricing?', *Working Paper* .

Merton, R. C. (1973), 'An intertemporal capital asset pricing model', *Econometrica: Journal of the Econometric Society* pp. 867–887.

Nieuwerburgh, S. V., Lustig, H. N. and Koijen, R. S. (2012), 'The cross-section and time-series of stock and bond returns', *Working Paper* .

Normandin, M., St-Amour, P. et al. (1998), 'Substitution, risk aversion, taste shocks and equity premia', *Journal of Applied Econometrics* **13**(3), 265–281.

Petkova, R. (2006), 'Do the fama–french factors proxy for innovations in predictive variables?', *The Journal of Finance* **61**(2), 581–612.

Skiadas, C. (2009), *Asset pricing theory*, Princeton University Press.

Weil, P. (1989), 'The equity premium puzzle and the risk-free rate puzzle', *Journal of Monetary Economics* **24**(3), 401–421.

Table 1: State Price Ratios

This table displays state price ratios from equation (2.26) at different values of relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS).

A. RRA > 1

	Relative Risk Aversion								
EIS	**1.01**	**1.10**	**1.25**	**1.5**	**2**	**3**	**5**	**10**	**25**
0.04	***1.05***	***1.05***	***1.05***	***1.05***	***1.05***	***1.05***	***1.04***	***1.03***	***1.00***
0.10	***1.05***	***1.05***	***1.05***	***1.05***	***1.05***	***1.04***	***1.03***	***1.00***	*0.92*
0.20	***1.05***	***1.05***	***1.05***	***1.05***	***1.04***	***1.03***	***1.00***	*0.94*	0.77
0.33	***1.05***	***1.05***	***1.05***	***1.04***	***1.03***	***1.00***	***0.95***	0.84	0.57
0.50	***1.05***	***1.05***	***1.04***	***1.03***	***1.00***	***0.95***	0.86	0.66	0.31
0.67	***1.05***	***1.04***	***1.03***	***1.00***	***0.95***	0.86	0.70	0.42	0.09
0.80	***1.05***	***1.03***	***1.00***	***0.95***	0.86	0.70	0.46	0.17	<.01
0.91	***1.05***	***1.00***	*0.93*	0.81	0.63	0.38	0.14	0.01	<.01
0.99	***1.00***	0.63	0.29	0.08	<.01	<.01	<.01	<.01	<.01
1.01	*1.11*	1.77	3.84	14.04	>100	>100	>100	>100	>100
1.10	*1.06*	1.11	1.21	1.40	1.85	3.25	10.06	>100	>100
1.25	*1.06*	*1.08*	1.12	1.20	1.36	1.76	2.94	10.59	>100
1.5	*1.05*	*1.07*	*1.09*	1.14	1.23	1.43	1.95	4.20	42.28
2	*1.05*	*1.06*	*1.08*	*1.11*	1.17	1.29	1.59	2.65	12.35
3	*1.05*	*1.06*	*1.07*	*1.09*	1.14	1.23	1.43	2.10	6.67
5	*1.05*	*1.06*	*1.07*	*1.09*	1.12	1.20	1.36	1.87	4.90
10	*1.05*	*1.06*	*1.07*	*1.08*	1.11	1.18	1.32	1.76	4.13
25	*1.05*	*1.06*	*1.07*	*1.08*	*1.11*	1.17	1.30	1.70	3.79

B. RRA < 1

	Relative Risk Aversion								
EIS	**0.04**	**0.10**	**0.20**	**0.33**	**0.50**	**0.67**	**0.80**	**0.91**	**0.99**
0.04	*1.05*	*1.05*	*1.05*	*1.05*	*1.05*	*1.05*	*1.05*	*1.05*	*1.05*
0.10	*1.06*	*1.06*	*1.06*	*1.06*	*1.06*	*1.05*	*1.05*	*1.05*	*1.05*
0.20	*1.07*	*1.06*	*1.06*	*1.06*	*1.06*	*1.06*	*1.06*	*1.05*	*1.05*
0.33	*1.08*	*1.08*	*1.07*	*1.07*	*1.07*	*1.06*	*1.06*	*1.06*	*1.05*
0.50	*1.11*	*1.10*	*1.10*	*1.09*	*1.08*	*1.07*	*1.06*	*1.06*	*1.05*
0.67	1.16	1.15	1.14	1.13	*1.11*	*1.09*	*1.07*	*1.06*	*1.05*
0.80	1.28	1.27	1.24	1.21	1.17	1.13	*1.10*	*1.07*	*1.05*
0.91	1.72	1.67	1.59	1.48	1.36	1.25	1.17	*1.10*	*1.06*
0.99	>100	>100	63.74	32.16	13.68	5.82	2.94	1.68	*1.11*
1.01	<.01	<.01	0.02	0.03	0.08	0.19	0.37	0.66	***1.00***
1.10	0.61	0.63	0.67	0.72	0.79	0.87	*0.94*	***1.00***	***1.05***
1.25	0.82	0.84	0.86	0.89	*0.93*	***0.97***	***1.00***	***1.03***	***1.05***
1.5	*0.91*	*0.92*	*0.93*	***0.95***	***0.97***	***1.00***	***1.02***	***1.04***	***1.05***
2	***0.95***	***0.96***	***0.97***	***0.98***	***1.00***	***1.02***	***1.03***	***1.04***	***1.05***
3	***0.98***	***0.98***	***0.99***	***1.00***	***1.01***	***1.03***	***1.04***	***1.05***	***1.05***
5	***0.99***	***0.99***	***1.00***	***1.01***	***1.02***	***1.03***	***1.04***	***1.05***	***1.05***
10	***1.00***	***1.00***	***1.01***	***1.01***	***1.02***	***1.03***	***1.04***	***1.05***	***1.05***
25	***1.00***	***1.00***	***1.01***	***1.02***	***1.02***	***1.03***	***1.04***	***1.05***	***1.05***

Table 2: VAR Results

y1 is the nominal log yield on a one-month treasury bill. Inflation is one-month log inflation. Relative y1 and relative inflation are the difference between current yields and inflation and average values over the past twelve months. y120 - y3 is the yield spread between 10-year and 3-month treasury bonds. rmrf is the excess return of the CRSP value weighted market return over the risk free rate. d - p is the log dividend-price ratio, calculated for the CRSP value-weighted market index using current prices and average dividends over the past twelve months. Results are for a 1-lag VAR of demeaned y1, inflation, relative y1, relative inflation, rmrf, and d-p. Coefficients for dependent variables y1 and inflation are reported. The other dependent variables are omitted for brevity. Bootstrapped standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

	1983-2012		**1927-2012**	
	(1)	**(2)**	**(3)**	**(4)**
	y1	**inflation**	**y1**	**inflation**
Lagged Variables				
y1	0.9639***	0.1939*	0.9741***	0.0631
	(0.0202)	(0.1003)	(0.0116)	(0.0773)
inflation	0.0314	0.0737	0.0102*	0.7762***
	(0.0297)	(0.1734)	(0.0062)	(0.0709)
relative y1	-0.0976**	0.1295	-0.1752***	0.5909***
	(0.0457)	(0.1585)	(0.0407)	(0.1599)
relative inflation	-0.0136	0.3268*	-0.003	-0.4554***
	(0.0281)	(0.1767)	(0.0056)	(0.0837)
y120 - y3	-0.0032	-0.002	-0.0062**	0.0014
	(0.0036)	(0.0155)	(0.0024)	(0.0122)
rmrf	0.0013*	0.0083*	0.0008**	0.0061*
	(0.0007)	(0.0042)	(0.0004)	(0.0034)
d - p	0.0001	0.0002	0.0000	-0.0002
	(0.0001)	(0.0005)	(0.0000)	(0.0003)
R-Squared	0.95	0.24	0.95	0.32

Table 3: Real Risk Free Rate News Covariance Deciles

Value-weighted decile portfolios are formed at the end of each month by sorting stocks based on covariance with risk free rate news over the past three years. The table reports betas with respect to risk free rate news, average size, and average excess returns for each portfolio. The table also reports results for time series regressions of excess returns on excess market returns (the CAPM regression) and excess market returns, the Fama-French size factor (smb), and the Fama-French value factor (hml) (the 3 Factor regression). Standard errors for the 10-1 portfolio difference are reported in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance. The sample is NYSE, AMEX, and NASDAQ common stocks.

A. 1985-2012

Decile	**Rf News Beta**	**Market Cap ($B)**	**Excess Return**	**CAPM Alpha**	**3 Factor Alpha**	**Factor Loadings (Betas)**		
						rmrf	**smb**	**hml**
1	-0.17	0.72	0.63%	-0.19%	-0.16%	1.27	0.61	-0.06
2	0.07	1.36	0.94%	0.24%	0.30%	1.10	0.22	-0.15
3	-0.04	1.94	0.87%	0.25%	0.23%	1.04	0.07	0.04
4	0.13	2.42	0.65%	0.06%	0.03%	1.00	-0.04	0.09
5	0.00	2.74	0.51%	-0.03%	-0.05%	0.94	-0.10	0.03
6	0.02	2.76	0.48%	-0.06%	-0.08%	0.93	-0.14	0.05
7	0.03	2.58	0.54%	-0.02%	-0.04%	0.97	-0.11	0.03
8	0.15	2.21	0.68%	0.06%	0.08%	1.04	-0.13	-0.07
9	0.14	1.69	0.61%	-0.06%	-0.04%	1.10	0.01	-0.06
10	0.41	0.85	0.21%	-0.62%	-0.44%	1.21	0.55	-0.47
10-1	0.58**	0.13**	-0.42%	-0.42%	-0.27%	-0.06	-0.07	-0.41***
	(0.23)	(0.06)	(0.33%)	(0.34%)	(0.34%)	(0.08)	(0.11)	(0.12)

B. 1929-2012

Decile	**Rf News Beta**	**Market Cap ($B)**	**Excess Return**	**CAPM Alpha**	**3 Factor Alpha**	**Factor Loadings (Betas)**		
						rmrf	**smb**	**hml**
1	-0.01	0.17	0.66%	-0.05%	-0.12%	1.15	0.52	-0.03
2	0.00	0.48	0.66%	0.04%	0.03%	1.04	0.20	-0.06
3	0.03	0.69	0.70%	0.13%	0.12%	0.99	0.08	-0.01
4	0.06	0.86	0.71%	0.15%	0.15%	0.96	0.02	0.00
5	0.01	0.98	0.60%	0.04%	0.02%	0.97	-0.03	0.06
6	0.03	1.05	0.56%	-0.01%	-0.03%	0.98	-0.03	0.09
7	0.06	1.08	0.58%	-0.01%	-0.02%	1.03	-0.08	0.08
8	0.06	1.05	0.56%	-0.07%	-0.10%	1.08	0.00	0.11
9	0.10	0.83	0.61%	-0.07%	-0.12%	1.15	0.04	0.17
10	0.11	0.38	0.58%	-0.18%	-0.27%	1.23	0.50	0.03
10-1	0.13	0.21***	-0.09%	-0.13%	-0.14%	0.07**	-0.02	0.05
	(0.09)	(0.02)	(0.18%)	(0.18%)	(0.18%)	(0.03)	(0.06)	(0.05)

Table 4: Equity Market and Bond Real Interest Rate Risk

rmrf is the excess return on the CRSP value weighted market portfolio. Decile 10-1 is returns to long-short portfolio representing the difference between the 10th and first riskfree rate news covariance portfolios, described in Table 2. 1-2 and 5-10 year bonds represent excess returns to treasury bonds of those durations, as calculated by CRSP. Correlations and betas with respect to riskfree rate news and average returns are reported for each return series. The price of beta is defined as average returns divided by beta. Standard errors are reported in parentheses. Standard errors for the price of beta are calculated using the delta method. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

A. 1985-2012

	rmrf	**Decile 10-1**	**1-2 Year Bonds**	**5-10 Year Bonds**
Rf News	0.04	0.14**	-0.14***	-0.03
Correlation	(0.05)	(0.05)	(0.05)	(0.05)
Rf News	0.11	0.58**	-0.04***	-0.04
Beta	(0.17)	(0.23)	(0.02)	(0.06)
Average	0.60%**	-0.42%	0.12%***	0.34%***
Excess Returns	(0.25%)	(0.33%)	(0.02%)	(0.09%)
Price of	5.35%	-0.72%**	-3.14%***	-9.70%
Beta	(10.57%)	(0.30%)	(0.64%)	(13.81%)

B. 1952-2012

	rmrf	**Decile 10-1**	**1-2 Year Bonds**	**5-10 Year Bonds**
Rf News	0.05	0.12***	-0.40***	-0.12***
Correlation	(0.04)	(0.04)	(0.03)	(0.04)
Rf News	0.10	0.30***	-0.12***	-0.10***
Beta	(0.08)	(0.09)	(0.01)	(0.03)
Average	0.55%***	-0.16%	0.09%***	0.16%***
Excess Returns	(0.16%)	(0.19%)	(0.02%)	(0.06%)
Price of	5.43%	-0.54%	-0.72%***	-1.57%***
Beta	(5.91%)	(0.46%)	-(0.12%)	-(0.13%)

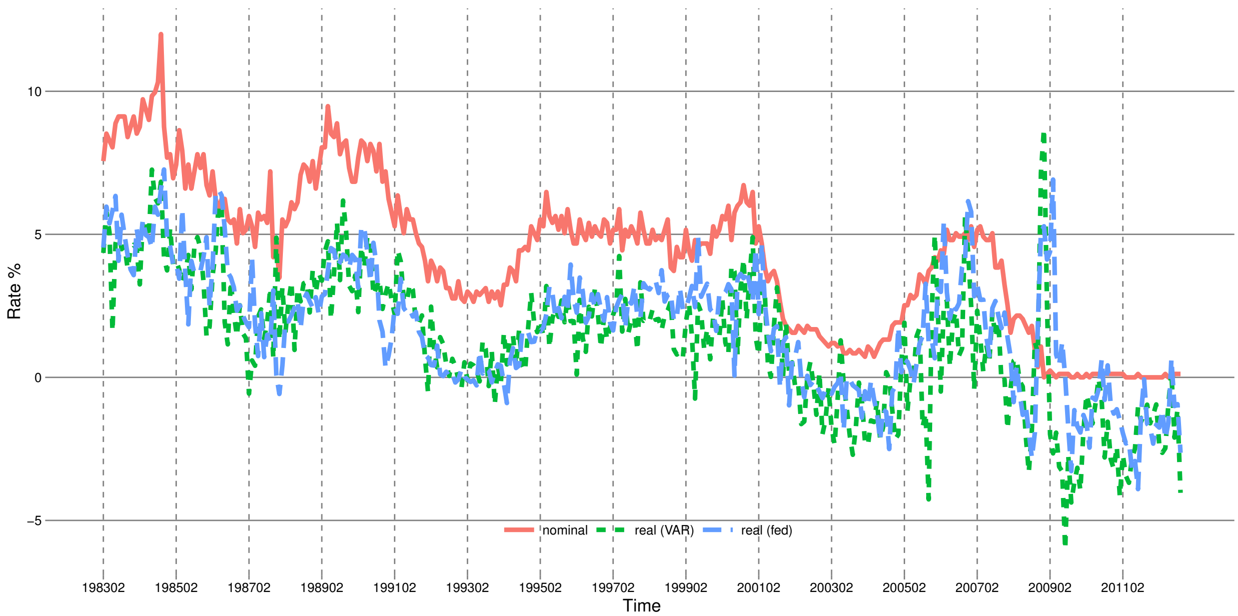

Figure 1: Risk Free Rates 1983-2012

The nominal risk free rate is the yield on a one-month nominal treasury bill. The real risk free rate is estimated using our VAR analysis. We also report the real risk free rate estimated by the Federal Reserve Bank of Cleveland.

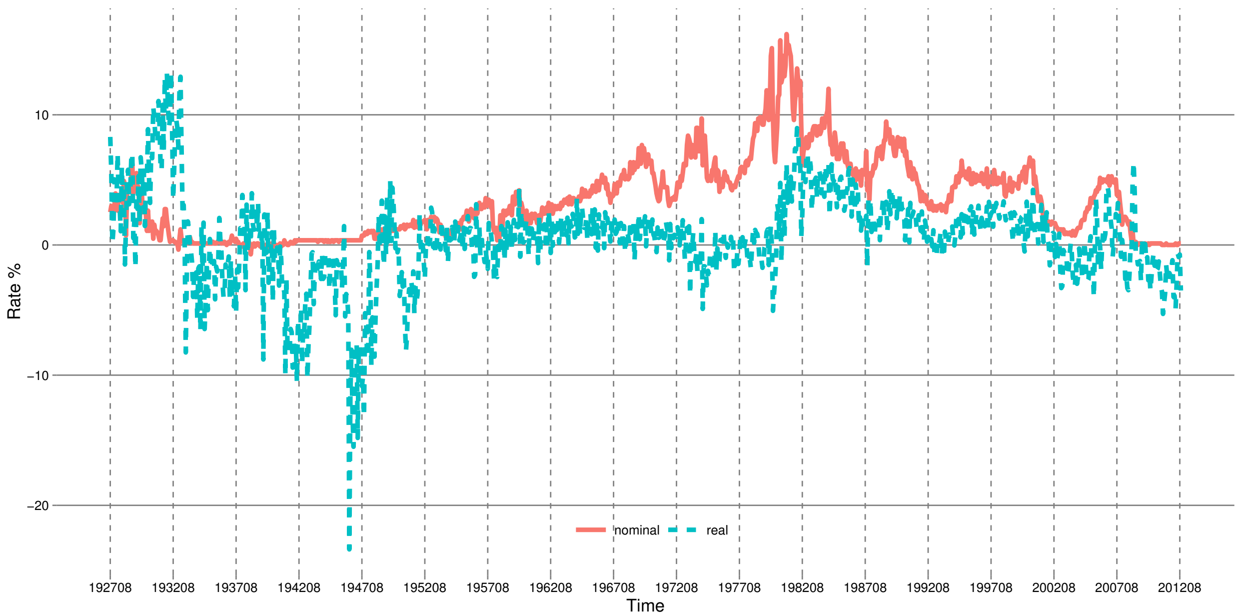

Figure 2: Risk Free Rates 1927-2012

The nominal risk free rate is the yield on a one-month nominal treasury bill. The real risk free rate is estimated using our VAR analysis.